

# Period-Colour and Amplitude-Colour Relations in Classical Cepheid Variables - VI. New Challenges for Pulsation Models

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## ABSTRACT

We present multiphase Period-Color/Amplitude-Color/Period-Luminosity relations using OGLE III and Galactic Cepheid data and compare with state of the art theoretical pulsation models. Using this new way to compare models and observations, we find convincing evidence that both Period-Color and Period-Luminosity Relations as a function of phase are dynamic and highly nonlinear at certain pulsation phases. We extend this to a multiphase Wesenheit function and find the same result. Hence our results cannot be due to reddening errors. We present statistical tests and the urls of movies depicting the Period-Color/Period-Luminosity and Wesenheit relations as a function of phase for the LMC OGLE III Cepheid data: these tests and movies clearly demonstrate nonlinearity as a function of phase and offer a new window towards a deeper understanding of stellar pulsation. When comparing with models, we find that the models also predict this nonlinearity in both Period-Color and Period-Luminosity planes. The models with ( $Z = 0.004, Y = 0.25$ ) fare better in mimicking the LMC Cepheid relations, particularly at longer periods, though the models predict systematically higher amplitudes than the observations.

**Key words:** Cepheids — distance scale.

## 1 INTRODUCTION

Classical Cepheids are the most important primary distance indicators within the Local Group thanks to their characteristic Period-Luminosity (PL) and Period-Luminosity-Color (PLC) relations. They are also currently used to calibrate secondary distance indicators and in turn to evaluate the Hubble constant ( $H_0$ , see e.g. Freedman et al. 2001; Saha et al. 2001). Any systematic effect on the Cepheid PL relations is expected to propagate on secondary distance indicators and, in turn, on the final evaluation of  $H_0$ . During the last decade there has been a very lively debate on the possible non-universality of Cepheid PL relations traditionally assumed to have the slope of the one derived in the Large Magellanic Cloud (see e.g. Freedman et al. 2001;

Bono et al 2008; Marconi 2009, and references therein). Unfortunately, in spite of many observational and theoretical efforts, there is not a conclusive answer to this question: nonlinear pulsation models predict a significant dependence both on metallicity and the helium content with final effects on the distance scale that can be higher than 10 % (Marconi 2009; Bono et al 2008, and references therein) and empirical tests that either find no metallicity effect or predict a metallicity dependence with an opposite sign with respect to the theoretically predicted one. However, spectroscopically based analyses by Romaniello et al. (2005, 2008) support the trend suggested by the pulsation model results.

Beyond the debated dependence on chemical composition there are both observational (e.g. Kanbur & Ngeow 2004; Ngeow et al. 2005, 2009) and theoretical (e.g. Caputo, Marconi, & Musella 2000; Marconi, Musella, & Fiorentino 2005; Kanbur et al. 2004) indications that the Cepheid PL relation is not always

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linear. Ngeow & Kanbur (2006a) have also shown that the effect on the inferred  $H_0$  of neglecting this possible nonlinearity can be of 1 – 2%, that combined with the predicted dependence on chemical composition can be important in the light of current efforts to push down errors on the  $H_0$  estimate to a few percent.

The Cepheid PL relation used for distance determination is evaluated at mean light: the mean over all pulsation phases. On the other hand recent studies by Kanbur & Ngeow (2004); Ngeow et al. (2005); Ngeow & Kanbur (2006b) have suggested that an innovative and promising approach to Cepheid pulsation is investigating the Period-Color (PC) and the PL relations as a function of phase. Indeed modern data on Cepheids are characterized by excellent phase coverage, particularly in the Magellanic Clouds. This offers a unique opportunity for deriving empirical multiphase PC and PL relations. This approach has already been adopted by Ngeow & Kanbur (2006b) on the basis of data from the OGLE II project for Magellanic Cepheids and from various literature sources for Galactic pulsators. These authors have demonstrated that the study of multi-phase relations can provide new insights into Cepheid pulsation. They find strong evidence that these observed PC/PL relations vary significantly with phase, are highly dynamic and strongly nonlinear at various pulsation phases usually located near minimum light. These relations offer a new way to constrain models and provide deeper insights into pulsation physics by comparing these observed multiphase PC/PL relations with those from theoretical models. In this paper we update our multiphase PC/PL relations using OGLE III LMC data and present a detailed comparison between theoretical and empirical multiphase PC and PL relations for the Milky Way and the Large Magellanic Cloud (LMC).

Simon et al. (1993), Kanbur & Ngeow (2004) and Ngeow & Kanbur (2006b) also showed the importance of amplitude-color (AC) relations. A flat PC relation at maximum/minimum light implies a relation between amplitude and color at minimum/maximum light. Kanbur & Ngeow (2004) found evidence of a change of slope in the PC relation at maximum light at a period of about 10 days and a corresponding change in the AC relation. Hence this paper also presents AC relations as a function of phase.

## 2 THE OBSERVED MULTIPHASE PC/PL RELATIONS

The data used in this study comes primarily from OGLE III and were corrected for reddening using the methods described in Ngeow et al. (2009). The data were fitted with a Fourier series like

$$V = A_0 + \sum_{k=1}^{k=N} A_k \sin(k\omega t + \phi_k),$$

where  $A_0$  is the mean magnitude,  $\omega = \frac{2\pi}{P}$ , and  $P$  is the period in days,  $t$  is the time of observation and  $N$  is the order of the fit. Because the phase coverage and the number of stars is large, the order of the Fourier fit was 8. The resulting Fourier fit was then used to infer properties at maximum/minimum/mean light. We performed a number of tests

which checked that the observed multiphase PC/AC/PL relations were independent of the order of the Fourier fit. The Fourier fits were rephased so that maximum light occurs at phase 0 in the V band, as described in Ngeow & Kanbur (2006b).

In what follows, short and long period Cepheids are demarcated by a period of 10 days. Our results are presented in table 1 and figures 1-8 using OGLE III data for the LMC results together with results for Galactic Cepheids using data from Ngeow & Kanbur (2006b). Following an anonymous referee's suggestions we present only figures 1 and 2 in the published version. Figures 3-8 are in the on-line version of the paper. However in the text that follows we do refer to all these figures.

In all these figures, short/long period observed data are represented by black and blue crosses respectively. Theoretical results are usually represented by colored symbols related to the mass of the models in question. These are noted on the figures. Some of our results pertain solely to the observed behavior, some solely to the theoretical relations and some to the comparison of both. However, to save space, we have superimposed the theoretical relations to the observed ones.

The results presented in figures 1-8 depict PC/AC/PL relations at maximum and minimum light. These figures broadly support the work of Ngeow & Kanbur (2006b) and clearly demonstrate the dynamic nature of the PC/PL relations as a function of phase. These figures, and the movies (see also the url addresses given below), provide clear evidence of a nonlinearity at phases close to minimum light - statistical tests are not needed. This nonlinearity is seen in PC/PL/AC relations: changes in one relation are reflected in the other. In particular, we see that at minimum light, the nonlinearity is marked, and moreover, it is sharp: see also Ngeow & Kanbur (2006b) and figures 5 and 6 here. That is the data strongly imply a significant change in the slope of both PC/AC/PL relations at a period of about 10 days.

Table 1 presents the results of  $F$  tests as outlined in Ngeow & Kanbur (2006b) and references therein for the PC and PL relations at maximum and minimum light for the LMC using OGLE III Cepheids. In this table, we present the slope and zero points for the long and short period regression lines, the dispersion, the number of data points used and the value of the  $F$  statistic. For the number of data points used here, an  $F$  value greater than about 3 indicates that the data are more consistent with the alternative hypothesis at the 95% confidence interval. We note that certain phases such as minimum light are strongly non-linear according to the  $F$  test whilst other phases (close to maximum light) are marginally linear. The combination of these phases again results in a dilution of the nonlinearity at mean light and suggests why this effect has not been observed before. Note that Ngeow et al. (2009) only presented the results of such statistical tests at mean light.

Our results support and extend the conclusions of Ngeow & Kanbur (2006b)

- There is a marked nonlinearity at a period of  $\log P \approx 1$  in the PC/PL/AC plots. The PL relation at maximum light is linear and is strongly nonlinear at minimum light.
- The PC plots show a great deal of structure with, possibly, changes of slope at other periods as well as at  $\log P \approx 1$ .
- There is a variation in dispersion of both PC/PL rela-

**Table 1.**  $F$  test results for PC/PL/PW relations at various phases

Period	Slope	ZP	$\sigma$	N
<i>LMCPC(max)</i>				
All	$0.241 \pm 0.018$	$0.296 \pm 0.012$	0.144	1625
$\log(P) < 1.0$	$0.319 \pm 0.026$	$0.251 \pm 0.016$	0.142	1517
$\log(P) > 1.0$	$-0.246 \pm 0.134$	$0.830 \pm 0.156$	0.159	108
$F$ -statistic is 14.87 (non-linear)				
<i>LMCPC(min)</i>				
All	$0.281 \pm 0.015$	$0.573 \pm 0.01$	0.117	1602
$\log(P) < 1.0$	$0.182 \pm 0.021$	$0.629 \pm 0.013$	0.114	1494
$\log(P) > 1.0$	$0.510 \pm 0.113$	$0.352 \pm 0.131$	0.134	108
$F$ -statistic is 22.255 (non-linear)				
<i>LMCPL<sub>V</sub>(max)</i>				
All	$-2.672 \pm 0.044$	$16.691 \pm 0.030$	0.351	1624
$\log(P) < 1.0$	$-2.574 \pm 0.064$	$16.635 \pm 0.039$	0.351	1516
$\log(P) > 1.0$	$-3.032 \pm 0.291$	$17.066 \pm 0.337$	0.344	108
$F$ -statistic is 2.71 (linear)				
<i>LMCPL<sub>V</sub>(min)</i>				
All	$-2.639 \pm 0.038$	$17.395 \pm 0.025$	0.298	1621
$\log(P) < 1.0$	$-2.883 \pm 0.053$	$17.535 \pm 0.033$	0.293	1513
$\log(P) > 1.0$	$-1.658 \pm 0.260$	$16.365 \pm 0.301$	0.307	108
$F$ -statistic is 24.87 (non-linear)				
<i>LMCPW(phase = 0.0)</i>				
All	$-3.279 \pm 0.020$	$15.921 \pm 0.013$	0.160	1629
$\log(P) < 1.0$	$-3.368 \pm 0.027$	$15.973 \pm 0.017$	0.151	1522
$\log(P) > 1.0$	$-2.437 \pm 0.195$	$14.976 \pm 0.226$	0.229	107
$F$ -statistic is 26.16 (non-linear)				
<i>LMCPW(phase = 0.25)</i>				
All	$-3.321 \pm 0.027$	$15.819 \pm 0.013$	0.152	1629
$\log(P) < 1.0$	$-3.252 \pm 0.027$	$15.780 \pm 0.016$	0.146	1522
$\log(P) > 1.0$	$-3.231 \pm 0.178$	$15.677 \pm 0.207$	0.209	107
$F$ -statistic is 6.97 (non-linear)				
<i>LMCPW(phase = 0.5)</i>				
All	$-3.275 \pm 0.0233$	$15.793 \pm 0.014$	0.172	1629
$\log(P) < 1.0$	$-3.199 \pm 0.030$	$15.750 \pm 0.019$	0.166	1522
$\log(P) > 1.0$	$-3.465 \pm 0.194$	$15.977 \pm 0.225$	0.228	107
$F$ -statistic is 6.28 (non-linear)				
<i>LMCPW(phase = 0.75)</i>				
All	$-3.329 \pm 0.028$	$15.921 \pm 0.019$	0.223	1629
$\log(P) < 1.0$	$-3.391 \pm 0.040$	$15.956 \pm 0.025$	0.222	1522
$\log(P) > 1.0$	$-3.121 \pm 0.199$	$15.708 \pm 0.230$	0.233	107
$F$ -statistic is 2.61 (linear)				

tions as a function of phase. The PC and PL relation at maximum light has a greater dispersion than at minimum light. Moreover the dispersion for a given phase also decreases at periods close to 10 days - at least for the LMC.

- Both Galactic and LMC PC relations at maximum light are flat but the OGLE III results again suggest that the Galactic PC relation is flat for  $\log P \geq 0.8$  whilst the LMC PC relations is flat for  $\log P \geq 1$ .

- We see that at minimum light, higher amplitude Cepheids are driven to redder colors when the PC relation is flat or flatter for those periods - as originally proposed

by Simon et al. (1993). For example, the AC(min) relation for Galactic Cepheids is one relation across the entire period range. However, the AC(max) relation is clearly two separate relations, demarcated by a sharp change at a period of 10 days with the longer period relation having a non-zero slope (Ngeow & Kanbur 2006b). This corresponds to a change in the slope of the LMC PC(max and min) relation at a period of 10 days. This is important because a number of authors (Keller et al. 2006) have noted correlations between mean color and amplitude. Since mean light relations are simply the average of the same relations as a function of phase,

then a correlation between mean color and amplitude has to be due to the correlation between amplitude and pulsation phases around minimum light. This, in turn, is caused by the interaction of the hydrogen ionization front (HIF) and photosphere (Simon et al. 1993; Kanbur et al. 2004, 2006). This again demonstrates why such a multiphase analysis is useful and demanded by the high quality data now available.

- Movies of the multiphase PC/PL relations, viewable at <http://www.oswego.edu/~kanbur/IRES2009/Cphase.mov>, <http://www.oswego.edu/~kanbur/IRES2009/Vphase.mov>, provide further strong evidence of the nonlinear nature of the PC/PL relation at phases around minimum light. Further studies of such movies is warranted since they seem to indicate the presence of a number of shocks and different behavior for Cepheids in different period ranges. These movies also suggest that a group of Cepheids mostly with periods around 10 days (perhaps the bump Cepheids) are the first to brighten subsequent to initial dimming after maximum light. While this may be due to the Hertzsprung progression, this offers an opportunity to study the Hertzsprung progression from a different perspective.

### 3 THE THEORETICAL SCENARIO

We selected various sets of nonlinear convective pulsation models computed with the code originally developed by Stellingwerf (1982); Bono & Stellingwerf (1994) and adapted to Cepheid pulsators by Bono, Marconi & Stellingwerf (1999). The selected models cover a range of chemical compositions, namely  $Z=0.02$ ,  $Y=0.31$ ;  $Z=0.02$ ,  $Y=0.28$ ;  $Z=0.01$ ,  $Y=0.26$ ;  $Z=0.004$ ,  $Y=0.25$ ;  $Z=0.008$ ,  $Y=0.25$ , considered representatives of Galactic and Magellanic Cepheids (see Marconi, Musella, & Fiorentino 2005; Bono et al 2008; Marconi 2009, for details). For each selected chemical composition and stellar mass we considered canonical models, that are models following the mass-luminosity (ML) relation derived by Bono et al (2000a) without taking into account either overshooting or mass-loss. The model bolometric light curves presented in Bono et al (2000b); Marconi, Musella, & Fiorentino (2005) are converted into magnitude and color variations using static stellar atmospheres (Castelli, Gratton, & Kurucz 1997a,b) and used to derive theoretical multiphase PC/PL relations in the various bands. Figures 1-8 display these theoretical relations. In each plot, a given symbol represents pulsation models at a given mass (and hence luminosity, through the adopted  $ML$  relation), composition and a range of effective temperatures.

In looking at just the theoretical PC relations, we see that some mass sequences are monotonic, some suffer a gradual change of slope whilst others are distinctly non-monotonic, and further, this behaviour is sometimes different (for a given mass sequence) at maximum and minimum light: for example, see figure 7 - the  $9M_{\odot}$ ,  $Y = 0.25$ ,  $Z = 0.004$  mass sequence.

Examples of the non-monotonic behaviour typically bracket a period of 10 days but in some cases, this behaviour also occurs at periods greater than 10 days.

### 4 THE MULTIPHASE PC/PL RELATIONS: THEORY VERSUS OBSERVATIONS

Inspection of Figures 1-8 suggests that:

- The models with  $Z = 0.004$ ,  $Y = 0.25$  fare the best in reproducing the LMC PC/PL/AC relations, particularly at long periods - see for example, figures 6-7, which compare the PC/PL relations at maximum/minimum light with model results. In particular note the comparison for PC relations at maximum/minimum light for periods in the range between  $1.2 < \log P < 1.5$ .

- The observed LMC AC relation displays a group of stars across a wide period range which are distinct from the main group eg. figure 7. These are modeled quite well by the 9-11 solar mass models with ( $Z = 0.004$ ,  $Y = 0.25$ ) composition. So it could be that the scatter in these plots is real and these stars are high mass stars with increasingly lower amplitude: perhaps because they are close to the edge of the instability strip. Certainly, the non-monotonic nature of the purely theoretical AC relations at maximum/minimum light would indicate the presence of such stars.

- It is also interesting to note that the LMC data are also quite nicely reproduced by models with  $Z = 0.02$   $Y = 0.31$ , whereas they disagree with models at  $Z = 0.02$   $Y = 0.28$ . This occurrence confirms the result that helium and metallicity effects tend to compensate each other.

- $V_{max}$  and  $V_{min}$  as a function of amplitude are systematically higher for models than for observations. This might be due to the sensitivity of model amplitudes to residual uncertainties in the treatment of turbulent convection.

- Figures 2-4, particularly the PC relation at maximum light, indicate that models with ( $Z = 0.02$ ,  $Y = 0.28$ ) fare better at reproducing the Galactic observations than models with ( $Z = 0.01$ ,  $Y = 0.26$ ).

- For Galactic Cepheids, the amplitude range covered by models is much wider than the observed one.

- For a given mass and periods shorter than 10 days, the theoretical PC/PL relation at maximum light has a greater positive slope than at minimum light (where the relations are almost flat). If the models can be regarded as doing a reasonable job of mimicking the observations, this shows why, in part, the observed PC/PL relations have a greater dispersion at maximum rather than at minimum light - see, for example figure 7.

### 5 THE MULTIPHASE WESENHEIT FUNCTION

A number of different formulations of the Wesenheit function exist in the literature. Madore and Freedman (1998) define it to be

$$W_V = V - 2.45(V - I),$$

whilst Udalski et al (1999), who adopt a slightly different extinction law, use the following definition,

$$W_I = I - 1.55(V - I).$$

Here the quantities  $V, I$  are the mean observed magnitudes in these bands.  $W_V$  and  $W_I$  can be shown to be reddening independent and for this reason the Wesenheit function is the preferred way to use the Cepheid PL relation

to estimate distances. It is of interest here to consider the linearity/nonlinearity of the Wesenheit function as defined by Udalski et al (1999).

If we phase both  $V$  and  $I$  observations relative to a common time origin and then, following a Fourier decomposition, rephase such that maximum  $V$  band brightness is called phase 0, then we can formulate a multiphase "Wesenheit-type" function as

$$W_{Iph} = I_{ph} - 1.55(V_{ph} - I_{ph}),$$

where  $V_{ph}$  and  $I_{ph}$  denote these quantities at the phase value  $ph$ . The results of the  $F$  test for nonlinearity applied to the Wesenheit function are presented in table 1 and figure 9. We see clearly that certain phases of the Wesenheit function, for example at maximum light, are strongly nonlinear. Since the reddening along the line of sight cannot vary due to pulsation phase, this result provides strong evidence against the hypothesis that previous statistical tests indicating nonlinearity are due to reddening errors. Previous work such as Koen et al. (2007) suggested that linear Wesenheit functions were due to nonlinearities in the PC and PL relations canceling out: the results presented here support this view.

Another nice demonstration of the nonlinearity of the Wesenheit function at certain phases is presented at the following url.

<http://www.oswego.edu/~kanbur/IRES2009/Wphase.mov>,

where we have 100 phase points, the  $y$  axis is the Wesenheit function and the  $x$  axis is  $\log P$ . Again we clearly see a dynamic  $W_I$  function which changes as a function of phase, not as much as the  $V$  and  $I$  band PL relations but nevertheless the nonlinearity at certain phases is clear and unambiguous and cannot be due to lack of data in certain period ranges. We get similar results we use the Madore and Freedman (1998) formulation.

It is our contention that the  $F$  test results presented in Table 1 provide very strong evidence for a change in the slope of the PL, PC and PW relations at a period of 10 days and provides evidence of the pulsation phases which are the most nonlinear.

## 6 CONCLUSIONS

Comparing observations and theory on the multiphase PC/PL/AC plane as described here is a powerful new way to constrain models and gain deeper insights into pulsation physics. The dynamic nature of the multiphase plots provides a new window into the inner workings of Cepheids and deserves more attention both observationally and theoretically. This can best be seen by viewing movies of these multiphase PC/PL relations available - see above for web links. The Wesenheit movie depicts the variation with phase of the Wesenheit function, defined by  $W = I - 1.55 * (V - I)$ . A number of authors have suggested that because previous work testing for possible nonlinearities in the Wesenheit function has yielded negative results, indicating nonlinearities in the PL/PC relations at mean light may be due to reddening errors (the Wesenheit function is reddening independent). Our movies indicate clear nonlinearities in the multiphase Wesenheit function implying that the cause of the nonlinearity cannot be due to reddening. The effect of

this nonlinearity on the extra-galactic distance scale and CMB independent estimates of  $H_0$  is a matter for debate.

It may be argued that a comparison of observations and theory in the multiphase PC/PL planes are another projection of the comparison of observed and theoretical light curves: even if the PL/PC/PW relations at mean light were linear, there is no similar expectation for linearity of the multiphase relations because of, for example, the Hertzsprung progression. This broad argument may be true but it still needs to be demonstrated and the demonstration of the dynamic nonlinear behavior of the PL and PW relations is one of the main results of this paper. The average of the multiphase relations does surely yield information about the mean light relations.

Does such a comparison provide additional insight over and above a comparison of light curves? We would argue that it does. For example, flat PC relations at maximum light yield information about the interaction of the photosphere and hydrogen ionization front - something which would be very difficult to probe by just a comparison of observed and theoretical light curves. Our multiphase comparisons suggest that the greatest nonlinearity occurs at minimum light: this information, when investigated in greater detail can provide a deeper understanding of Cepheid pulsation over and above a comparison of observed and theoretical light curves. This will be the topic of future work.

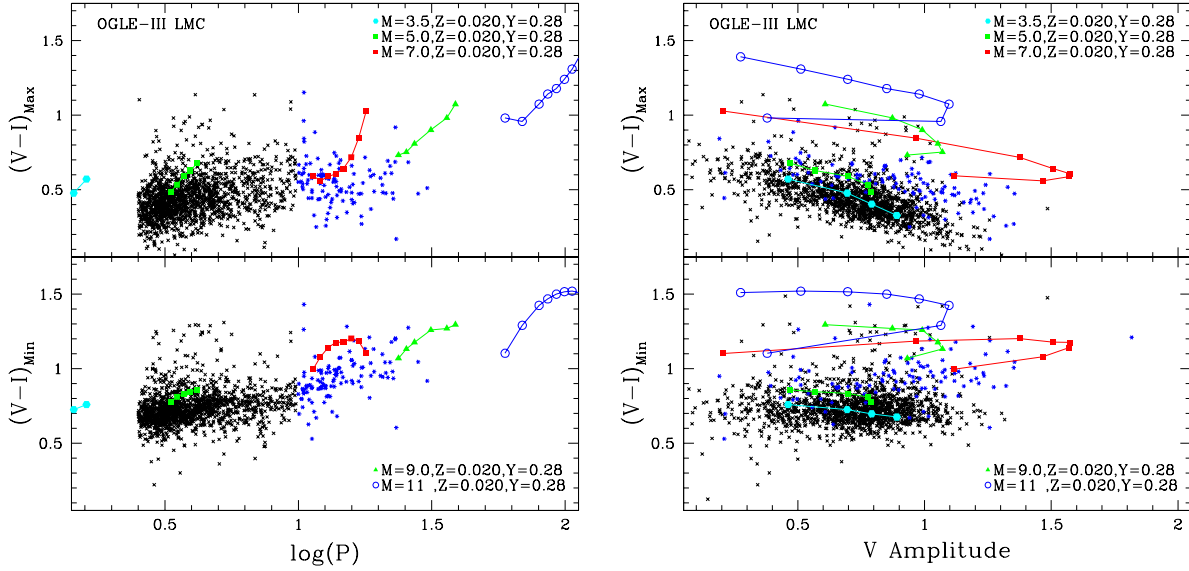
In any case the results presented in this paper represent an important challenge for theoreticians seeking a deeper understanding of Cepheid pulsation.

## ACKNOWLEDGMENTS

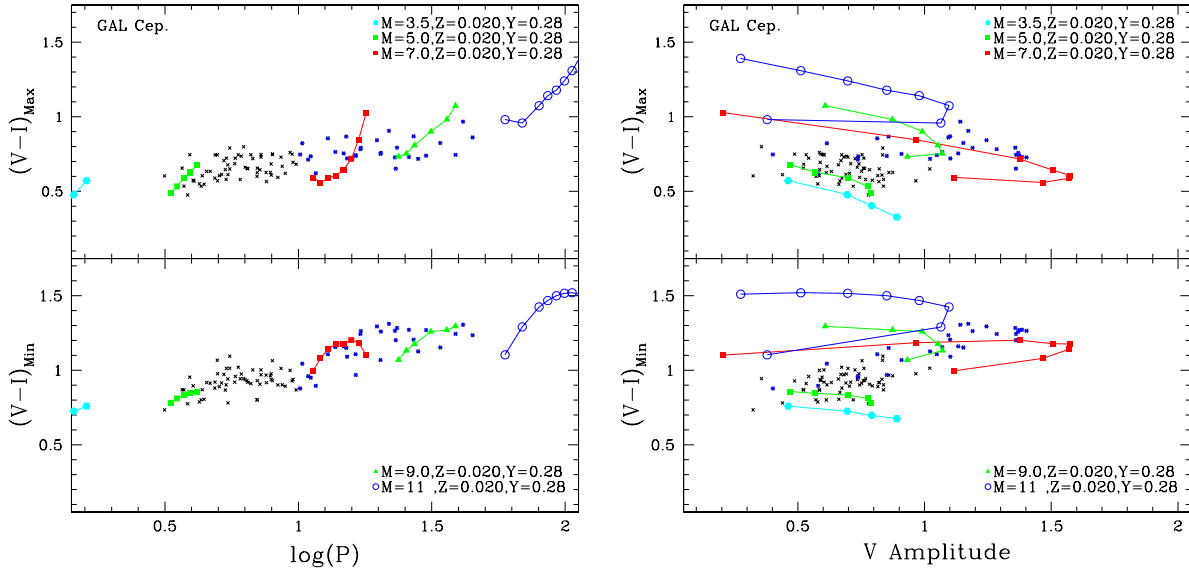
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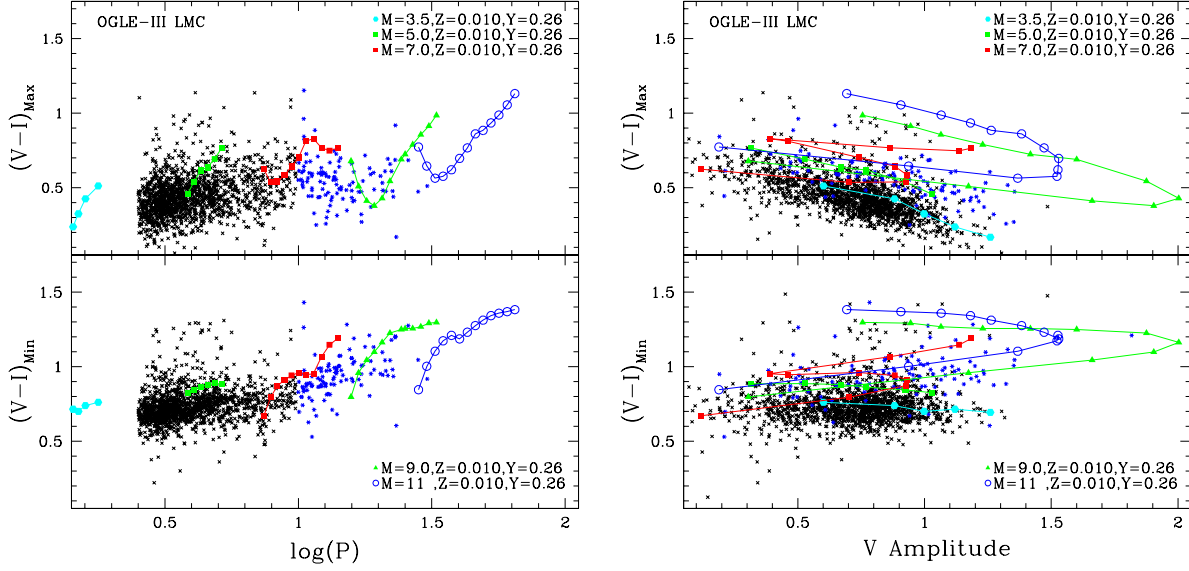
**Figure 1.** Multiphase PC/AC relations for reddening corrected OGLE III LMC Cepheid data and theoretical models with  $Z = 0.02$ ,  $Y = 0.28$ .



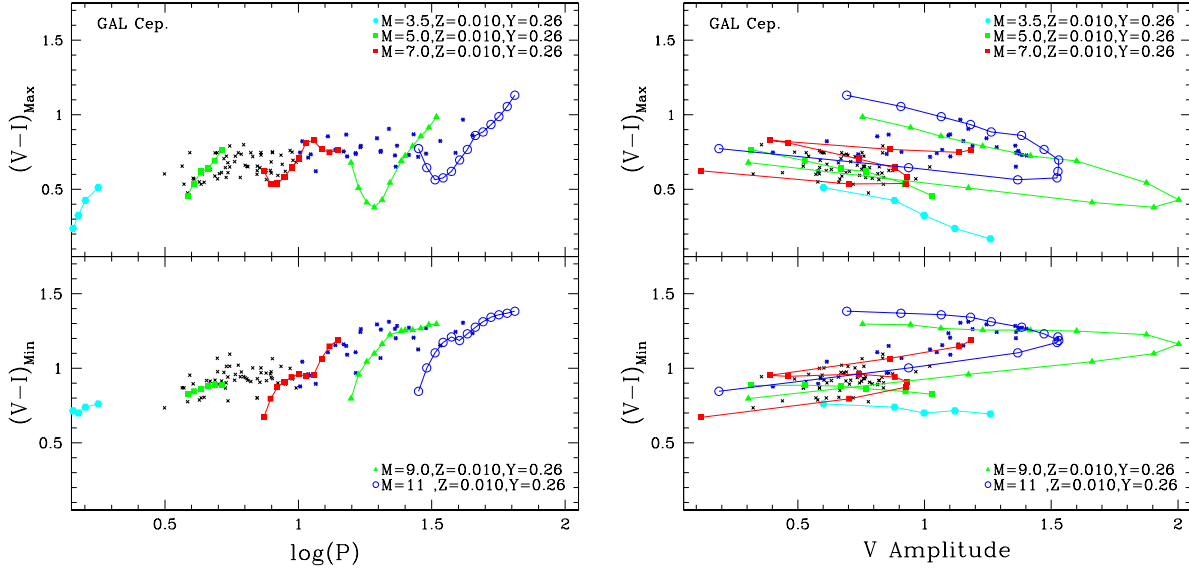
**Figure 2.** Multiphase PC/AC relations for reddening corrected Galactic Cepheid data and theoretical models with  $Z = 0.02$ ,  $Y = 0.28$ .

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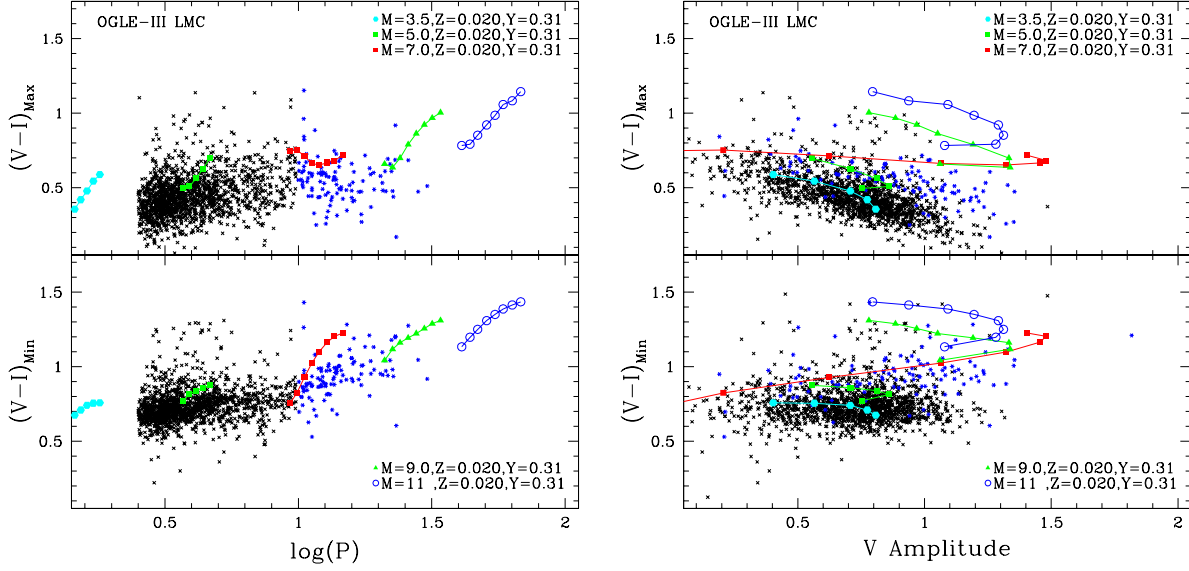
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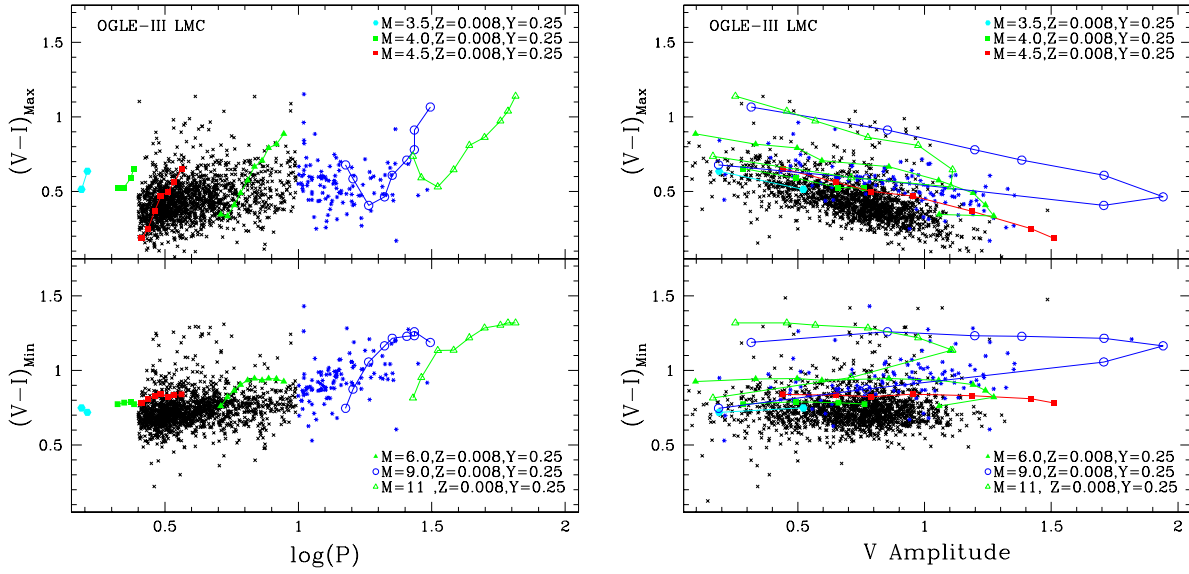
**Figure 3.** Multiphase PC/AC relations for reddening corrected OGLE III LMC Cepheid data and theoretical models with  $Z = 0.01$ ,  $Y = 0.26$ .



**Figure 4.** Multiphase PC/AC relations for reddening corrected Galactic Cepheid data and theoretical models with  $Z = 0.01$ ,  $Y = 0.26$ .

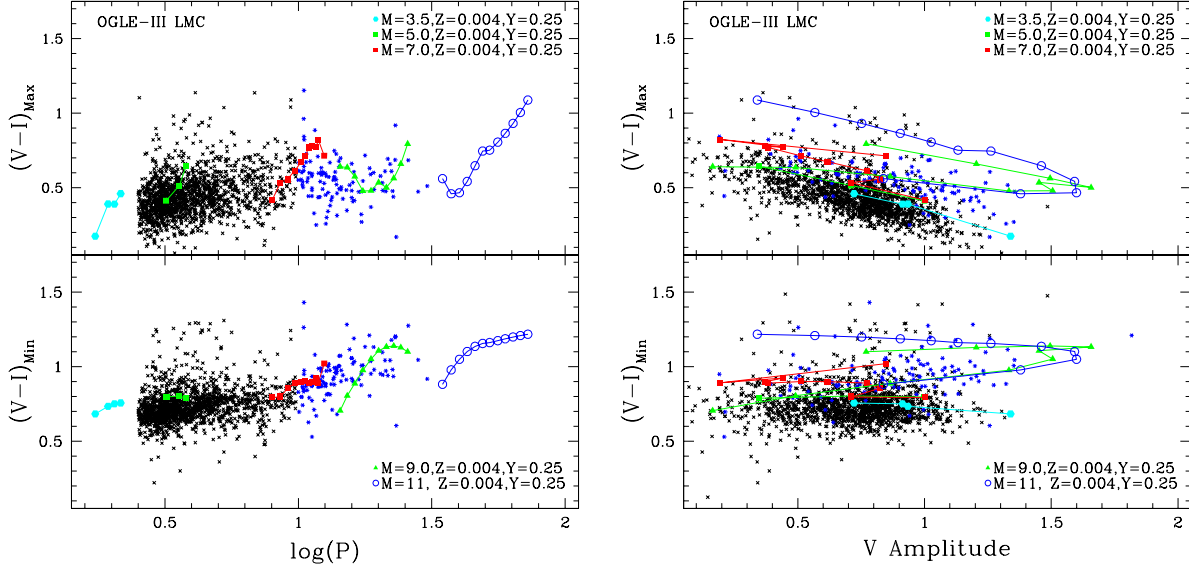


**Figure 5.** Multiphase PC/AC relations for reddening corrected OGLE III LMC Cepheid data against theoretical models with  $Z = 0.02$ ,  $Y = 0.31$ .

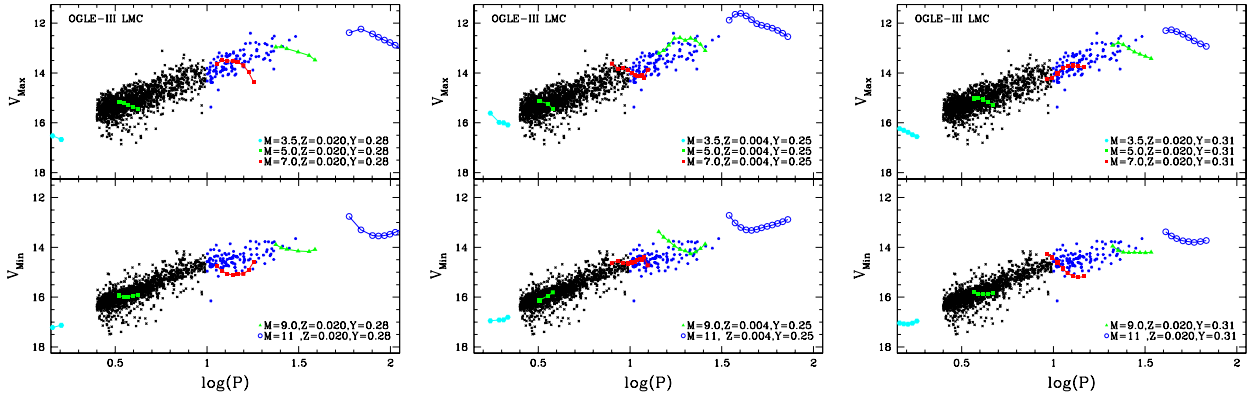


**Figure 6.** Multiphase PC/AC relations for reddening corrected OGLE III LMC Cepheid data against theoretical models with  $Z = 0.008$ ,  $Y = 0.25$ .

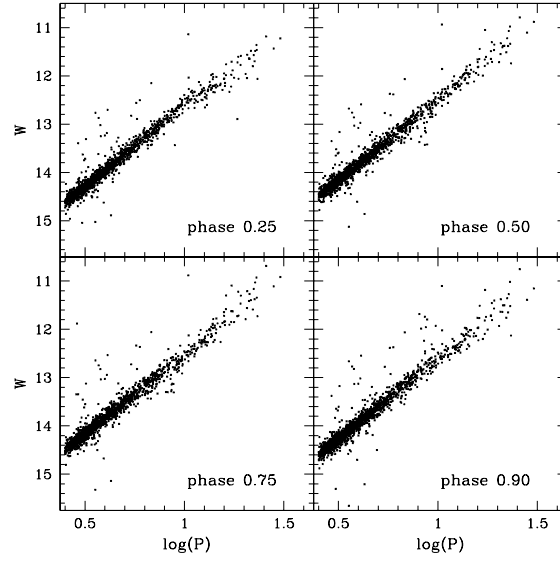




**Figure 7.** Multiphase PC/AC relations for reddening corrected OGLE III LMC Cepheid data against theoretical models with  $Z = 0.004$ ,  $Y = 0.25$ .



**Figure 8.** Multiphase PL relations for reddening corrected OGLE III LMC Cepheid data against theoretical models with  $(Z = 0.02, Y = 0.28)$ ,  $(Z = 0.004, Y = 0.25)$ ,  $(Z = 0.02, Y = 0.31)$ .



**Figure 9.** Multiphase Wesenheit function for OGLE III LMC Cepheid data showing clear nonlinearities at certain phases.

